Introduction to Hall Effect

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The Measurement of Conductivity provides information about the number of charge carriers' N' and their mobility. This much information is not complete for any crystalline material.

Because it does not give information about the nature of charge carriers.

This information can be gathered by the measurement of Hall Voltage.

Hall effect depends upon the acceleration of a moving charge by a magnetic field.

The force on the charge, q, which produces this acceleration is given by;

$$\mathbf{F} = q\, \mathbf{v} \times \mathbf{B}$$

*Where **v** is the velocity of the charge '**q**' and **B** is the magnetic induction. Thus the force depends upon the magnitude and sign of the charge.*

Moreover, this **F** ,is perpendicular to both **v** and **B**.

Lets take **v** to be drift velocity of the charge under the effect of the electric field **E**.

Sine the charge may either be positive or negative, the **v** would be either parallel or antiparallel to **E**.

In case of Metal and ionic solids this distinction is not so important as there exist only electron as charge carriers.

But in case of semiconductors, it becomes an important factor.

Let **E** = **Ex** and **B** = **Bz............................[1]**

Consider a Cube. Z axis being perpendicular to x-y plane. Electric field applied along x direction, and Magnetic field along Z direction.

Thus the force on the charge, q, due to the electric field would be **q.Ex .**

Because of the drift velocity , the charge experiences a force in the magnetic field, which changes their straight line trajectory in the x-y plane to curved path.

The drift velocity can be written as **v** = **i** vx + **j** vy +**k** vz, ...[2]

So the force on the charge resulting from its motion in the magnetic field **B** is

F = q. **v** x **B,** **as B = k**Bz using equation **1**

we have **F = q. (i** vx + **j** vy +**k** vz **)x (k**Bz**)**

Thus **F** = q. (**i**vyBz – **j** vx Bz)............................[3]

- Since along the electric field E_x, the velocity of the charge is v_x, thus the magnetic field produces the force along y-direction, *perpendicular to both x and z direction*, on these charges. The will make near face –ive and far face to be +ive.
- Such accumulation of charges set up a potential difference or field along this direction viz E_y. Thus the electron experiences an additional force qE_y. This force would be in a direction opposite to the force exerted by the magnetic field.
- Thus the net force along the y direction must be zero,
- Taking only y component,

$$q \mathbf{E}_y + q \mathbf{v}_x \times \mathbf{B}_z = 0$$

Or

$$qE_y = - q \mathbf{v}_x \times \mathbf{B}_z \quad \text{as } \mathbf{v}_x = (qE_x \tau)/ m^*$$

Thus

$$q E_y = - (q^2 \tau/ m^*) \mathbf{E}_x \times \mathbf{B}_z$$

$$\text{or } E_y = - (q \tau/ m^*) \mathbf{E}_x \times \mathbf{B}_z \text{ ----- [4]}$$

Ey = - (q τ/ m*) **E**x X **B**z ,   or

Ey / **E**x X **B**z = - (q τ/ m*),   This defines the mobility, **μ** = - (q τ/ m*) ---- [5]

Thus, Replacing Ex = (jx / σ) , We have **E**y = - (q τ/ m*) **E**x X **B**z

or **E**y = - (q τ/ m*) (jx / σ) X **B**z , or   **E**y = - (q τ/ m*) (jx **B**z)/ σ

or **E**y = - (q τ/σ m*) (jx **B**z) ,   But σ = N (q2 τ/ m*) -------- [6]

Thus we have **E**y = - (1/ Nq) (jx **B**z)

Or **E**y = - R(jx **B**z) Where R = 1/Nq --------------[7]

This ,R, is called Hall Constant,

Since we have taken the conduction due to electron –ive value R, For Hole it will be +ive

Now since the mobility **μ** = q τ/ m* , using 5,6,7 we have

IRI σ = μ .... [8]

- This gives a relation in between hall constant, conductivity and Mobility of charge carriers.

Thus, Hall effect gives following information;

- The sign of the current carrying charges,
- Number of charge carriers per unit volume which can be calculated from the magnitude of hall constant R ,
- The mobility of charge carriers which can be measured directly.
- If both electron and holes contribute to conductivity, R can either be +ive or -ive , depending upon the relative densities and mobility of the charge careers.