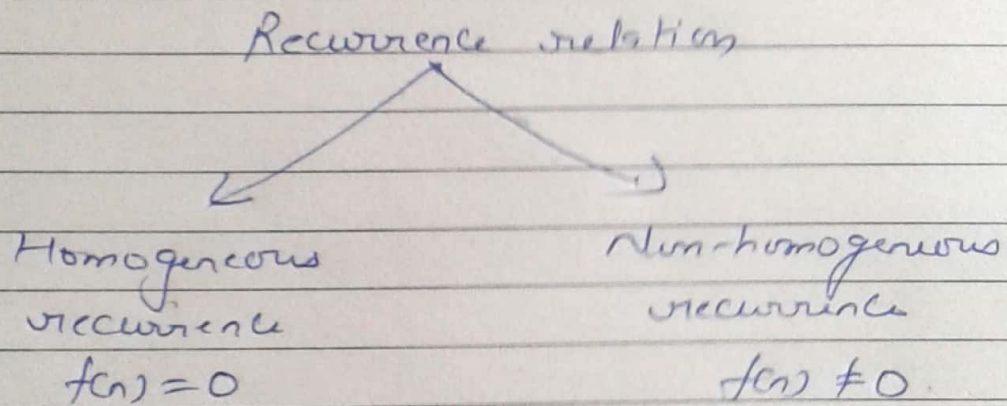


Recurrence relation:-

A recurrence relation for the sequence $\{f_n\}$ is a formula that expresses f_n in terms of one or more previous terms of the sequence such as f_0, f_1, \dots, f_{n-1} for all integers n .

for example $f_n = 2f_{n-1} - f_{n-2}$



Homogeneous recurrence

A k^{th} -order linear relation is a homogeneous recurrence relation if $f(n) = 0 \forall n$.

Non-Homogeneous recurrence

A k^{th} order linear relation is a non-homogeneous recurrence relation if $f(n) \neq 0 \forall n$.

For example $a_n = a_{n-1} + a_{n-2}$ is a homogeneous relation.

$b_n = b_{n-1} + 2$ is a non-homogeneous recurrence relation,

Linear recurrence relation

A recurrence relation of the form

$$a_0 f_n + a_1 f_{n-1} + a_2 f_{n-2} + \dots + a_k f_{n-k} = f(n)$$

where a_i 's are constants and is called a linear recurrence relation with constant coefficient (U)

The recurrence relation (U) is known as a k^{th} order recurrence relation,

Here k^{th} -order means that each term in the sequence depends only on the k previous terms,

Numericals:-

Q.1 What is the characteristic equation of $Q(k) + 2Q(k-1) - 3Q(k-2) - 6Q(k-4) = 0$

Sol:- Given recurrence relation is of fourth order,

the characteristic equation of $Q(k) + 2Q(k-1) - 3Q(k-2) - 6Q(k-4) = 0$ is $x^4 + 2x^3 - 3x^2 - 6 = 0$.

Q. Solve the Fibonacci sequence $\{f_n\}$ defined by
initial conditions $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$ with the
 $f_0 = 0$ and $f_1 = 1$

Sol:- $f_n = f_{n-1} + f_{n-2}$

Characteristic equation corresponding to the given
difference equation is

$$x^2 - x - 1 = 0$$

on solving the quadratic equation, we get

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{1 \pm \sqrt{5}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$d_1 = \frac{1 + \sqrt{5}}{2}, \quad d_2 = \frac{1 - \sqrt{5}}{2}$$

General solution is

$$f_n = A_1 d_1^n + A_2 d_2^n$$

using initial conditions

$$f_0 = 0$$

then, $f_0 = A_1 d_1^0 + A_2 d_2^0$

$$0 = A_1 + A_2 \quad \text{--- (1)}$$

Again $f_1 = 1$

$$f_1 = A_1 d_1' + A_2 d_2'$$

$$1 = A_1 \left(\frac{1+\sqrt{5}}{2} \right)' + A_2 \left(\frac{1-\sqrt{5}}{2} \right)'$$

$$1 = A_1 \left(\frac{1+\sqrt{5}}{2} \right) + A_2 \left(\frac{1-\sqrt{5}}{2} \right) \quad \text{--- (2)}$$

on solving equation (1) & (2), we get =

from equation (1) we have

$$A_2 = -A_1$$

on substituting $A_2 = -A_1$ in equation (2) we get

$$1 = A_1 \left(\frac{1+\sqrt{5}}{2} \right) - A_1 \left(\frac{1-\sqrt{5}}{2} \right)$$

$$1 = A_1 \left[\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2} \right]$$

$$1 = \frac{2\sqrt{5}}{2} A_1$$

$$A_1 = \frac{1}{\sqrt{5}} \quad \text{then} \quad A_2 = -\frac{1}{\sqrt{5}}$$

Hence
$$f_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$