Discrete Mathematics

M.Sc Mathematics IVth Semester

Recurrence relation:-

A recurrence relation for the sequence $\{f_n\}$ is a formula that express f_n in terms of one or more previous terms of the sequence such as $f_0, f_1, \ldots f_{n-1}$ for all integers n.

for example $f_n = 2f_{n-1} - f_{n-2}$

Recurrence relation

Homogeneous recurrence	Non-homogeneous recurrence
$f(n) = 0$	$f(n) \neq 0$

Homogeneous recurrence

A K^{th}-order linear relation is a homogeneous recurrence relation if $f(n) = 0$ $\forall n$,

Non-Homogeneous recurrence

A k^{th} order linear relation is a non-homogeneous recurrence relation if $f(n) \neq 0$ $\forall n$.

For example $a_n = a_{n-1} + a_{n-3}$ is a homogeneous relation.

$b_n = b_{n-1} + 2$ is a non-homogeneous recurrence relation.

Linear recurrence relation

A recurrence relation of the form

$$a_0 f_n + a_1 f_{n-1} + a_2 f_{n-2} + \cdots + a_k f_{n-k} = f(n) \quad \text{--- (1)}$$

where a_i's are constants and is called a linear recurrence relation with constant coefficients.

The recurrence relation (1) is known as a k^{th} order recurrence relation.

Here k^{th}-order means that each term in the sequence depends only on the k previous terms.

Numericals:-

Q.1 What is the characteristic equation of $Q(K) + 2Q(K-1) - 3Q(K-2) - 6Q(K-4) = 0$

Sol:- Given recurrence relation is of fourth order.

The characteristic equation of $Q(K) + 2Q(K-1) - 3Q(K-2) - 6Q(K-4) = 0$

is $r^4 + 2r^3 - 3r^2 - 6 = 0$.

Q. Solve the Fibonacci sequence $\{f_n\}$ defined by $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$ with the initial conditions $f_0 = 0$ and $f_1 = 1$

Sol:- $f_n = f_{n-1} + f_{n-2}$

Characteristic equation corresponding to the given difference equation is

$$r^2 - r - 1 = 0$$

on solving the quadratic equation we get

$$\alpha = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\alpha = \frac{1 \pm \sqrt{5}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\alpha_1 = \frac{1+\sqrt{5}}{2}, \quad \alpha_2 = \frac{1-\sqrt{5}}{2}$$

General solution is

$$f_n = A_1 \alpha_1^n + A_2 \alpha_2^n$$

using initial conditions

$$f_0 = 0$$

then, $$f_0 = A_1 \alpha_1^0 + A_2 \alpha_2^0$$

$$0 = A_1 + A_2 \quad \text{— (1)}$$

Again $f_1 = 1$

$$f_1 = A_1 \alpha_1^1 + A_2 \alpha_2^1$$

$$1 = A_1 \left(\frac{1+\sqrt{5}}{2}\right)^1 + A_2 \left(\frac{1-\sqrt{5}}{2}\right)^1$$

$$1 = A_1 \left(\frac{1+\sqrt{5}}{2}\right) + A_2 \left(\frac{1-\sqrt{5}}{2}\right) \quad -(2)$$

on solving equation (1) & (2), we get-

from equation (1) we have

$$A_2 = -A_1$$

on substituting $A_2 = -A_1$ in equation (2)
we get

$$1 = A_1 \left(\frac{1+\sqrt{5}}{2}\right) - A_1 \left(\frac{1-\sqrt{5}}{2}\right)$$

$$1 = A_1 \left[\cancel{\frac{1}{2}} + \frac{\sqrt{5}}{2} - \cancel{\frac{1}{2}} + \frac{\sqrt{5}}{2}\right]$$

$$1 = \frac{\cancel{2}\sqrt{5}}{\cancel{2}} A_1$$

$$A_1 = \frac{1}{\sqrt{5}} \quad \text{then} \quad A_2 = \frac{-1}{\sqrt{5}}$$

Hence $$f_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

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